

Theoretical update of B -Mixing and Lifetimes

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We review the current status of theoretical predictions for mixing quantities and lifetimes in the B -sector. In particular, due to the first non-zero measurement of the decay rate difference in the neutral B_s -system, $\Delta\Gamma_s/\Gamma_s = 17.6\% \pm 2.9\%$ by the LHCb collaboration and very precise data for τ_{B_s} from TeVatron and LHCb our theoretical tools can now be rigorously tested and it turns out that the Heavy Quark Expansion works in the B -system to an accuracy of at least 30% for quantities like Γ_{12} , which is most sensitive to hypothetical violations of quark hadron duality. This issue that gave rise in the past to numerous theoretical papers, has now been settled experimentally. Further data will even allow to shrink this bound. For total inclusive quantities like lifetimes the compliance is even more astonishing: $\tau_{B_s}^{\text{LHCb}}/\tau_{B_d}^{\text{HFAG}} = 1.001 \pm 0.014$ is in perfect agreement with the theory expectation of $\tau_{B_s}/\tau_{B_d} = 0.996 \dots 1.000$.

Despite the fact that the new data show no deviations from the standard model expectations, there is still some sizable room for new physics effects. Model-independent search strategies for these effects are presented with an emphasis on the interconnection with many different observables that have to be taken into account. In that respect a special emphasis is given to the large value of the di-muon asymmetry measured by the D0 collaboration.

1 Introduction

Recently^a several hints for new physics effects in the flavour sector, see e.g. ^{1,2,3,4}, triggered a lot of interest. In particular there were several independent experimental indications ^{5,6,7,8,9,10} for a large CP violating phase in B_s -mixing, which would be in clear contradiction to the tiny standard model (SM) expectation ^{11,12,13,14,15}.

Unfortunately the huge expectations in spectacular new physics effects were not confirmed by precise measurements at LHCb or by follow-up measurements at the TeVatron, e.g. ^{16,17,18,19}. The current disappointment about this situation can be nicely visualized with Fig.(1). ^b

^aBefore the *Lepton Photon* Conference in August 2011.

^b A recent review on the implications of this new experimental results on several new physics models can be found in ²¹.

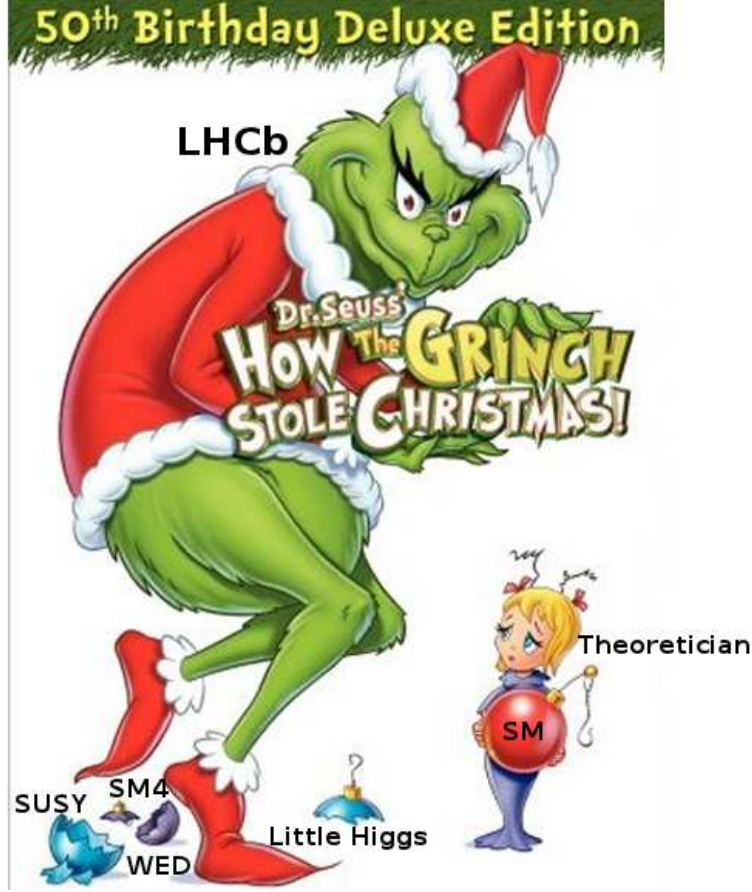


Figure 1: How the new LHCb results might appear at first sight. Previous measurements at TeVatron raised big expectations in large new physics effects, which were unfortunately not confirmed by more precise measurements at LHCb. It is, however, important to note that we gained many theoretical insights by the first measurement of $\Delta\Gamma_s$ and by other very precise measurements. This will be elaborated in this paper.

One of the main aims of this talk is to emphasize the fact that we now have for the first time a measurement of a non-zero value of the decay rate difference in the neutral B_s -system, $\Delta\Gamma_s$ ¹⁶. This quantity raised a lot of interest in the last thirty-five years and several open questions related to the theoretical determination of $\Delta\Gamma_s$ have now been answered experimentally. Another important point is the fact that there is still some ample space for new physics effects in B -mixing. To distinguish possible new physics effects from hadronic uncertainties, however, a considerably higher theoretical precision for questions like penguin pollution is now necessary, compared to what would have been needed in the case of a huge mixing phase.

In Section 2 we give a short introduction to the mixing formalism and to its experimental evidence. We also briefly touch the calculational tools, in particular the Heavy Quark Expansion (HQE). In Section 3 historic attacks to the HQE and their experimental resolution are described. Therefore we discuss theoretical arguments in favour of the HQE as well as the missing charm puzzle, the lifetimes of the Λ_b -baryon and the B_s -meson, the decay rate differences $\Delta\Gamma_d$ and $\Delta\Gamma_s$ and semi leptonic CP asymmetries. This chapter contains the main results: the proof of the applicability of the HQE to determine Γ_{12} . Model independent investigations of possible new physics effects in mixing are presented in Section 4, while Section 5 concentrates on new physics contributions and its constraints to the absorptive part of the mixing diagram, Γ_{12} . Such effects were discussed in order to explain the large central value of the di-muon asymmetry measured by the D0 collaboration. Since it turned out experimentally that new physics effects in B -mixing are not huge, investigations of the remaining space for effects beyond the standard model require

now a much better theoretical control of e.g. penguin contributions, this is discussed in Section 6. Finally we conclude.

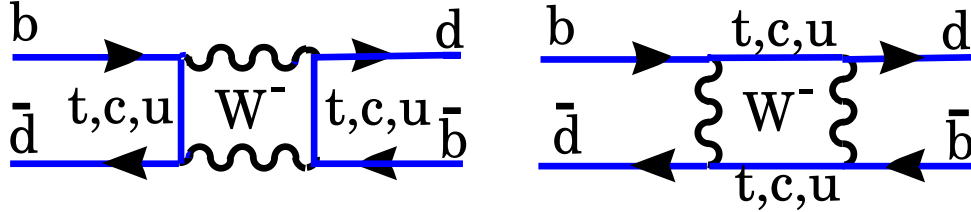
2 Mixing Formalism

2.1 Basic mixing formulae

The time evolution of a decaying particle $B(t) = \exp[-im_B t - \Gamma_B/2t]$ with mass m_B and lifetime $\tau_B = 1/\Gamma_B$ can be written as a differential equation. In the case of a two-state system (e.g. the neutral meson $B_q = (\bar{b}q)$ and its anti-particle $\bar{B}_q = (b\bar{q})$) one expects a diagonal mass matrix \hat{M}^q and a diagonal decay rate matrix $\hat{\Gamma}^q$:

$$i \frac{d}{dt} \begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix} = \left(\hat{M}^q - \frac{i}{2} \hat{\Gamma}^q \right) \begin{pmatrix} |B_q(t)\rangle \\ |\bar{B}_q(t)\rangle \end{pmatrix}. \quad (1)$$

It is known since a long time (e.g. ²²) that due to weak interactions transitions like $B_{d,s} \rightarrow \bar{B}_{d,s}$ are possible in the neutral B -system. They are triggered by the so-called *box diagrams*.



The box diagrams lead to off-diagonal elements in the mass matrix \hat{M}_q (denoted by M_{12}^q) and to off-diagonal elements in the decay rate matrix $\hat{\Gamma}_q$ (denoted by Γ_{12}^q). M_{12}^q corresponds to the part of the box-diagram, where the internal quarks are off-shell (e.g. the internal top-quark), while Γ_{12}^q denotes the part where the internal quarks are on-shell (in the diagrams shown above this can only happen for the up- and charm-quark). Both M_{12}^q and Γ_{12}^q can be complex due to their dependence on CKM-elements.

Diagonalization of \hat{M}^q and $\hat{\Gamma}^q$ gives mass eigenstates $B_{q,H}$ (H=heavy) and $B_{q,L}$ (L=light) with the masses M_H^q , M_L^q and the decay rates Γ_H^q , Γ_L^q , see e.g.²³:

$$B_{q,H} := p B_q - q \bar{B}_q \quad , \quad B_{q,L} := p B_q + q \bar{B}_q \quad \text{with } |p|^2 + |q|^2 = 1. \quad (2)$$

In the limit of no CP violation in mixing, the heavy mass eigenstate is CP-odd and the light one CP-even. Theoretical calculations of the box diagrams (M_{12}^q and Γ_{12}^q) can be related to three observables, expressed in terms of $|M_{12}^q|$, $|\Gamma_{12}^q|$ and the relative phase $\phi_q = \arg(-M_{12}^q/\Gamma_{12}^q)$, which describes CP-violation in mixing; the individual phases of M_{12}^q and Γ_{12}^q are unphysical:

- **Mass difference:**

The mass difference between the heavy and the light mass eigenstates of the neutral B_q meson is given by the modulus of the dispersive part of the box diagram.

$$\Delta M_q := M_H^q - M_L^q = 2|M_{12}^q| \left(1 - \frac{1}{8} \frac{|\Gamma_{12}^q|^2}{|M_{12}^q|^2} \sin^2 \phi_q + \dots \right). \quad (3)$$

In the standard model $|\Gamma_{12}^q|/|M_{12}^q|$ is of the order of 5 per mille^{11,12} for the B_q -system. Therefore the corrections in Eq.(3) are completely negligible. Since M_{12}^q describes the virtual part of the box diagram, it is sensitive to heavy internal particles like the top-quark, but also to hypothetical new states like SUSY-particles, KK-states, So, the mass difference is expected to be very sensitive to new physics effects.

- Decay rate difference:

The decay rate difference between the heavy and the light mass eigenstates of the neutral B_q meson is given by the modulus of the absorptive part of the box diagram.

$$\Delta\Gamma_q := \Gamma_L^q - \Gamma_H^q = 2|\Gamma_{12}^q| \cos \phi_q \left(1 + \frac{1}{8} \frac{|\Gamma_{12}^q|^2}{|M_{12}^q|^2} \sin^2 \phi_q + \dots \right). \quad (4)$$

In the standard model ϕ_s is close to zero^{11,12}, so the cosine gives to a good approximation a value of one. Since Γ_{12}^q describes the on-shell part of the box diagram it is sensitive to light (with a total mass below M_B) internal particles, like the up- and charm-quark. As we do not expect sizable new physics contributions to CKM-favoured tree-level decays like $b \rightarrow c\bar{c}s$, it seems reasonable to assume almost no (i.e. below the hadronic uncertainties) new physics effects in Γ_{12}^s . Therefore the only way for new physics to affect $\Delta\Gamma_s$ is the phase ϕ_s . In the standard model ϕ_s is close to zero, so new physics can only lower $\Delta\Gamma_s$ compared to its standard model value²⁴. We will challenge, however, the assumption that Γ_{12}^q can not be affected by new physics in Section 5.

- Flavour specific /semi leptonic CP asymmetries:

A flavour specific decay $B_q \rightarrow f$ is defined by the property

- The decays $\bar{B}_q \rightarrow f$ and $B_q \rightarrow \bar{f}$ are forbidden.

To simplify the analysis of untagged decays one often demands in addition that no direct CP violation occurs in the decay, i.e. $|\langle f|B_q\rangle| = |\langle \bar{f}|\bar{B}_q\rangle|$. Examples of such a decay are $B_s \rightarrow D_s^- \pi^+$ or the semi leptonic decay $B_q \rightarrow X l \nu$ - hence also the name semi leptonic CP asymmetry. The asymmetry is defined as

$$\begin{aligned} a_{sl}^q &\equiv a_{fs}^q = \frac{\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow \bar{f})} = -2 \left(\left| \frac{q}{p} \right| - 1 \right) \\ &= \left| \frac{\Gamma_{12}^q}{M_{12}^q} \right| \sin \phi_q = \text{Im} \frac{\Gamma_{12}^q}{M_{12}^q} = \frac{\Delta\Gamma_q}{\Delta M_q} \tan \phi_q. \end{aligned} \quad (5)$$

In the standard model these asymmetries are suppressed by the small values of $|\Gamma_{12}^q|/|M_{12}^q|$ and ϕ_q . Measuring a sizable value for the semi leptonic CP asymmetries would be a clear indication for new physics, provided that we have a precise control over the theoretical framework. Theoretical arguments in favour of this control and the experimental proof of this control, which is now available, will be discussed in detail below.

The phenomenon of particle-antiparticle mixing is a macroscopic quantum effect, which is by now well established in several systems of neutral mesons.

1955 K^0 -system: Mixing in the neutral K -system was theoretically developed in 1955 by Gell-Mann and Pais²³. Based on that framework the phenomenon of *regeneration* was predicted in the same year by Pais and Piccioni²⁵. Experimentally regeneration was confirmed in 1960²⁶. A huge lifetime difference between the two neutral K -mesons was established already in 1956²⁷.

1986 B_d -system: Mixing in the B_d -system was found 1986 by UA1 at CERN²⁹ (UA1 attributed the result however to B_s mixing) and 1987 by ARGUS at DESY²⁸. The large result for the mass difference ΔM_d can be seen as the first clear hint for an (at that time) unexpected

large value of the top quark mass^{30 c}. For the decay rate difference currently only upper bounds are available, see³² for the most recent and most precise bound.

2006/12 B_s -system: The large mass difference in the B_s -system was established by the CDF collaboration at TeVatron³³. In 2012 the LHCb Collaboration measured for the first time a non-vanishing value of the decay rate difference in the B_s -system¹⁶.

2007 D^0 -system: Here we have several experimental evidences (BaBar, Belle, Cleo, CDF, E791, E831) for values of $\Delta\Gamma/\Gamma$ and $\Delta M/\Gamma$ at the per cent level, but we still do not have a single measurement with a statistical significance of more than five standard deviations. The combination of the data³⁴ shows, however, unambiguously that D mesons mix.

A more detailed discussion of the experimental evidence for mixing can be found e.g. in³⁵. Because of several reasons, mixing observables seem to be very well suited for the search of hypothetical new physics effects. First, these observables are experimentally established - as summarized above. Next, in the standard model the box diagrams are strongly suppressed, because of being fourth order in the weak interaction. So even small new physics effects might give comparable and therefore detectable contributions. Finally, hadronic effects are well under control in the B mixing observables. This will be elaborated below.

2.2 Theoretical determination of M_{12}

Calculating the virtual part of the box diagrams with internal up-, charm- and top-quark and using the unitarity of the 3x3 CKM matrix one finds that the by far dominant contribution is given by the top-quark. It reads

$$M_{12}^q = \frac{G_F^2}{12\pi^2} (V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B. \quad (6)$$

The evaluation of the 1-loop box-diagram gives the so-called Inami-Lim function $S_0(x_t)$ ³⁶ with $x_t = m_t^2/M_W^2$. Perturbative QCD corrections to the box-diagrams are denoted by $\hat{\eta}_B$ ³⁷, they turned out to be ample. In the end one also has to determine the non-perturbative matrix element of a four quark operator Q (the four external legs of the box diagram) switched in between the states of the physical B and \bar{B} mesons. For historical reasons this matrix element is parameterized in terms of a bag parameter B_{B_q} and a decay constant f_{B_q} :

$$\langle \bar{B}_q | Q | B_q \rangle = \frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q}^2. \quad (7)$$

$$Q = \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha \cdot \bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\beta \quad (8)$$

α and β denote colour indices. The bag parameter and the decay constant have to be determined with non-perturbative methods like lattice QCD or QCD sum rules.

Formally one has performed in Eq.(6) an operator product expansion (OPE), by integrating out the heavy W -boson and the heavy top-quark. Corrections to that expansion are expected to be of the order m_b^2/M_W^2 , i.e. smaller than four per mille and thus far below the hadronic uncertainties.

Comparing experiment (HFAG³⁴) with theory¹¹ one sees a very nice agreement, but also the fact that the theoretical error is considerably larger than the experimental one. The theory error

^cTo avoid a very large value of the top quark mass, also different new physics scenarios were investigated, in particular a scenario with a heavy fourth generation of fermions and a top quark mass of the order of 50 GeV, see e.g. ³¹.

is currently dominated by the hadronic parameters.

$$\Delta M_d^{\text{SM}} = 0.543 \pm 0.091 \text{ ps}^{-1}, \quad \Delta M_d = 0.507 \pm 0.004 \text{ ps}^{-1} \quad (9)$$

(ALEPH, CDF, D0, DELPHI, L3,

$$\Delta M_s^{\text{SM}} = 17.30 \pm 2.6 \text{ ps}^{-1}, \quad \Delta M_s = 17.69 \pm 0.08 \text{ ps}^{-1} \quad (10)$$

OPAL, BABAR, BELLE, ARGUS, CLEO),
(CDF, D0, LHCb).

For the theory prediction in Eq.(9) and Eq.(10) taken from¹¹ a very conservative average¹ of lattice determinations for the matrix element in Eq.(7) was taken as an input^d. In particular

$$f_{B_s} = 231 \pm 15 \text{ MeV} \quad (11)$$

was used for the decay constant^e. Recently several new evaluations of the decay constant were performed^{40,41,42}. With this new numbers one obtains:

Reference	f_{B_s}	ΔM_s
HPQCD (1110.4510)	$225 \pm 4 \text{ MeV}$	$16.4 \pm 1.0 \text{ ps}^{-1}$
Fermilab/MILC (1112.3051)	$242 \pm 10 \text{ MeV}$	$19.0 \pm 1.8 \text{ ps}^{-1}$
CKMfitter new average (1203.0238)	$229 \pm 6 \text{ MeV}$	$17.1 \pm 1.7 \text{ ps}^{-1}$

(12)

For the first two results for ΔM_s in Eq.(12) only the value and the error of the decay constant was changed compared to Eq.(10). For the third number also a new average for the bag parameter was taken into account. In view of the relative large difference of the central values of the decay constants (17 MeV) determined in^{41,42} we prefer to stay currently with our conservative prediction given in Eq.(10).

Here a further reduction of the theoretical errors and a future reduction of the difference between distinct evaluations will be very desirable, because the mass differences give important bounds on fits⁴³ of the unitarity triangle and also to searches for new physics effects.

2.3 Theoretical determination of Γ_{12}

The theoretical determination of Γ_{12}^q is much more complicated than the one of M_{12}^q , sketched in the previous subsection. Γ_{12}^q is sensitive to real intermediate states like the up- and the charm-quark. Thus one can not integrate out at once all particles inside the loop of the box diagram, one has to follow instead a two step procedure:

1. OPE I: Integrate out the heavy W boson. This is similar to the case of M_{12}^q . Again, corrections to that approximation are expected to be at most at the per mille level.
2. OPE II: Perform an expansion in inverse powers of the heavy quark mass, the heavy quark expansion (HQE)^{44,45,46,47,48,49,50}.

Then the expression for Γ_{12}^q can be written in the following form:

$$\Gamma_{12}^q = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \dots\right) + \left(\frac{\Lambda}{m_b}\right)^5 \left(\Gamma_5^{(0)} + \dots\right) + \dots \quad (13)$$

Λ is a mass scale, which is expected to be of the order of Λ_{QCD} . Its precise value has, however, to be determined by explicit calculation. As will be shown below, for the B_s -system the expansion

^dFor a comparison of different ways to average the lattice results, see e.g. the discussion in³⁸.

^eFor more recent averages, see e.g.^{19,39}.

parameter Λ/m_b is a little smaller than $1/5$. The $\Gamma_i^{(j)}$'s are products of perturbatively calculable Wilson coefficients C_k and non-perturbative matrix elements of operators with a mass dimension equal or larger than six. Such a matrix element is also parameterized in terms of the decay constant f_B and a bag parameter B_k , i.e. $\Gamma_i^{(j)} \propto f_B^2 \sum C_k B_k$.

First estimates of $\Gamma_3^{(0)}$ were made starting from 1977 on, see e.g. ^{51,52,53,54,55,56}. At that time no effective Hamiltonian and no non-perturbative determinations of the decay constant and the bag parameters were available. Since 1977 clearly the determination of $\Gamma_3^{(0)}$ improved a lot. The subleading $1/m_b$ -corrections $\Gamma_4^{(0)}$ were determined for the first time in 1996 ⁵⁷ for B_s mesons, they turned out to be quite large. The NLO-QCD corrections $\Gamma_3^{(1)}$ for the leading CKM-structure in $\Delta\Gamma_s$ were calculated already in 1998 ⁵⁸. In 2003 this result was extended to the subleading CKM-structure by two independent groups ^{13,14}. These subleading terms give, however, the dominant contribution to the semi leptonic CP asymmetries and $\Delta\Gamma_d$, while they are a small, but non-negligible contribution to the decay rate difference $\Delta\Gamma_s$. In 2006 ¹² several theoretical improvements were incorporated in the theoretical determination of Γ_{12}^s , like an use of different quark mass schemes or summing up large logarithms of the form $m_c^2/m_b^2 \ln m_c^2/m_b^2$ to all orders (following ⁵⁹) as well as a change of the operator basis. Calculating Γ_{12} to the order Λ^4/m_b^4 one gets

$$\Gamma_{12}^q = C\langle Q \rangle_q + C_S\langle Q_S \rangle_q + \tilde{C}_S\langle \tilde{Q}_S \rangle_q + \sum_{i=1,2,3} \left(C_{R_i}\langle R_i \rangle_q + \tilde{C}_{R_i}\langle \tilde{R}_i \rangle_q \right), \quad (14)$$

with Wilson coefficients C , C_S , \tilde{C}_S and C_R and matrix elements of the operator Q appearing in M_{12} (defined in Eq.(8)) and of two new operators

$$Q_S = \bar{q}_\alpha(1 + \gamma_5)b_\alpha \cdot \bar{q}_\beta(1 + \gamma_5)b_\beta, \quad (15)$$

$$\tilde{Q}_S = \bar{q}_\alpha(1 + \gamma_5)b_\beta \cdot \bar{q}_\beta(1 + \gamma_5)b_\alpha. \quad (16)$$

R_i is an acronym for different operators of dimension seven, appearing in Γ_4 , which can be found e.g. in ¹². In ⁵⁷ it was noted that Q , Q_S , \tilde{Q}_S are not independent, they are related via

$$R_0 = Q_S + \alpha_1 \tilde{Q}_S + \frac{1}{2}Q. \quad (17)$$

The coefficients have the form $\alpha_i = 1 + \mathcal{O}(\alpha_s(m_b))$ and R_0 is of order $1/m_b$. So, to leading order in $1/m_b$ the quantity Γ_{12}^q can be expressed in terms of two independent operators, e.g. (Q, Q_S) , denoted as the *old basis* or (Q, \tilde{Q}_S) , denoted as the *new basis*.

$$\text{old basis: } \Gamma_{12}^q = C^{\text{old}}\langle Q \rangle_q + C_S^{\text{old}}\langle Q_S \rangle_q + C_{R_0}^{\text{old}}\langle R_0 \rangle_q + \sum_{i=1,2,3} \left(C_{R_i}\langle R_i \rangle_q + \tilde{C}_{R_i}\langle \tilde{R}_i \rangle_q \right). \quad (18)$$

$$\text{new basis: } \Gamma_{12}^q = C^{\text{new}}\langle Q \rangle_q + \tilde{C}_S^{\text{new}}\langle \tilde{Q}_S \rangle_q + C_{R_0}^{\text{new}}\langle R_0 \rangle_q + \sum_{i=1,2,3} \left(C_{R_i}\langle R_i \rangle_q + \tilde{C}_{R_i}\langle \tilde{R}_i \rangle_q \right). \quad (19)$$

In principle both expressions are absolutely equivalent. But, since the bag parameters of the different operators are not known with the same precision the choice of the basis has a sizable effect on the resulting accuracy in the determination of Γ_{12}^q . The best known matrix element is the one of the operator Q . This operator also arises in the mass difference ΔM_q and there are many non-perturbative determinations on the market. For the matrix element of Q_S or \tilde{Q}_S much less calculations are available, while for the matrix element of R_0 only vacuum insertion approximation is used - this corresponds to an uncertainty of 50%! Looking now at the sizes of the coefficients in Eq.(18) and Eq.(19) one finds ¹²

- $C_{R_0}^{\text{new}} < C_{R_0}^{\text{old}}$: In the new basis the coefficient of the operator R_0 is considerably smaller than in the old basis - this favours the new basis, because the matrix element of R_0 has a very large uncertainty.
- In the old basis C_5^{old} is the dominant coefficient, in the new basis the coefficient C^{new} of the best known operator is dominant - this favours the new basis.
- In the old basis, there is an accidental strong numerical cancellation in the coefficient C^{old} , which can lead to an overestimate of the theoretical uncertainty in the old basis.
- In the new basis the by far dominant contribution to the theory prediction for $\Delta\Gamma_q/\Delta M_q$ is absolutely free of non-perturbative uncertainties. This is not the case in the old basis, which is therefore disfavoured.

Thus we will use the new basis until very precise lattice values for all arising operators and the α_s/m_b -corrections will be available. In 2007 also the Wilson coefficients of the $1/m_b^2$ corrections, i.e. $\Gamma_5^{(0)}$ have been determined⁶⁰. Their size seems to be reasonably small. Since for some of the arising dimension eight operators even vacuum insertion approximation is not applicable, we do not include these terms in our numerics.

3 HQE under attack

Performing the HQE implicitly assumes that the sum over all possible exclusive final states, which is necessary to determine the total lifetime, is equal to the sum over all possible final state quarks. This assumption is called quark hadron duality, see e.g.⁶¹ or ⁶² for the application to decays of heavy hadrons. For total decay rates one obtains in the framework of the HQE

$$\Gamma = \Gamma_0 + \left(\frac{\Lambda}{m_b}\right)^2 \Gamma_2 + \left(\frac{\Lambda}{m_b}\right)^3 \Gamma_3 + \left(\frac{\Lambda}{m_b}\right)^4 \Gamma_4 + \dots \quad (20)$$

The leading term Γ_0 describes the decay of the free b-quark and is therefore the same for all b-hadrons. It depends on the fifth power of the b-quark mass, so depending on what quark mass scheme one is using, one gets very different results. Therefore it is advantageous to look at ratios of lifetimes of different b-hadrons. For different mesons this ratio starts with contributions of the order Λ^3/m_b^3 (for baryon vs. meson lifetime there are already corrections of the order of Λ^2/m_b^2), which look very similar to the contributions of Γ_{12}^q ^f. Comparing lifetime predictions with measurements provides a test of the assumption of quark hadron duality. Although one should keep in mind that for Γ_{12}^q one needs a stronger assumption, because less intermediate states (only the ones that are common to B_q and \bar{B}_q) contribute to the sum. To draw some definite conclusion on Γ_{12}^q also a direct determination of e.g. $\Delta\Gamma_q$ is mandatory, which is now available for B_s mesons¹⁶!

In the last 20 years regularly some discrepancies between experiment and theoretical predictions assuming quark hadron duality have been arising, which always triggered a lot of theoretical interest. Below I will discuss the following topics:

- In the mid 90ies the *missing charm puzzle* received a lot of attention, see e.g.⁶⁴ for a mini-review. The measured average number of charm-quarks per b-decay turned out to be lower than theoretically predicted: $n_c^{\text{Exp.}} < n_c^{\text{SM}}$. This discrepancy was also related to a mismatch of the experimental and theoretical values for the semi leptonic branching ratio. A possible solution was the violation of quark hadron duality, see e.g.^{65,66}.

^f A more detailed comparison of the theoretical determination of lifetime differences of different b-hadrons and of Γ_{12}^q is given e.g. in⁶³.

- From the beginning of the 90ies the values for the lifetime of the Λ_b baryon was measured to be much shorter, than the lifetime of the B_d mesons, while theory predicted them to be of similar size. This was also attributed to a failure of local duality in inclusive non leptonic heavy flavour decays, see e.g. ⁶⁷.
- Before 2003 the values of τ_{B_s}/τ_{B_d} were smaller than 0.95. Here theory predicts almost exactly one. This would also hint to a violation of quark hadron duality or to new physics effects in the lifetimes.
- In 2010 ^{7,8} and 2011 ⁶ the D0 collaboration announced an unexpected large value of the di-muon asymmetry. It turned out that assuming new physics in M_{12}^q is not sufficient to explain the large central value, one also needs large deviations of Γ_{12}^q from its standard model value, either via a violation of quark hadron duality or via new physics effects, see e.g. ⁶⁸.

In the following we will show that the above puzzles disappeared or that the solution via a violation of quark hadron duality is not viable anymore.

3.1 Theory arguments in favour of the HQE

In this section I will give some theoretical arguments in favour of the validity of the HQE. These arguments clearly represent no proof - for that an exact solution of QCD would have to be compared with the HQE, which is beyond the scope of the current paper.

1. A pragmatic starting point is to calculate corrections within the HQE in all possible “directions”, in order to test the convergence of the expansion. Splitting up the contributions to $\Delta\Gamma_s$ in a leading term (no perturbative QCD corrections, vacuum insertion approximation for the bag parameters and leading term in the HQE) and subleading corrections due to the use of lattice values for the bag parameters, due to perturbative QCD corrections and due to subleading HQE corrections one gets ⁶⁹:

$$\begin{aligned}\Delta\Gamma_s &= \Delta\Gamma_s^0 \left(1 + \delta^{\text{Lattice}} + \delta^{\text{QCD}} + \delta^{\text{HQE}}\right) \\ &= 0.142 \text{ ps}^{-1} (1 - 0.14 - 0.06 - 0.19) .\end{aligned}\tag{21}$$

Explicit calculation shows that the corrections are at most 19%, which seems to be a reasonable value and indicates no breakdown of the theory. In particular the expansion parameter of the HQE is now determined to be $0.19 \approx 1/5$. So no signal for a breakdown of the HQE is found by an explicit calculation^g.

2. As mentioned above (see ⁶³ for more details) the theoretical expressions for lifetime ratios of mesons resemble the expression for Γ_{12}^q .

$$\frac{\tau_{B_1}}{\tau_{B_2}} - 1 = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \dots\right) + \left(\frac{\Lambda}{m_b}\right)^5 \left(\Gamma_5^{(0)} + \dots\right) + \dots .\tag{22}$$

Moreover lifetimes are expected to be insensitive to new physics effects ^h. Comparing theory and experiment for $\tau(B^+)/\tau(B_d)$ one gets a very nice agreement ¹¹. Again, no sign

^gThis statement is of course much more solid than statements based on phase space arguments and simple power counting, see e.g. the discussion in ⁶⁹.

^hSizable new physics effect in lifetimes correspond to sizable new physics effects in the dominant decay modes, which have not been seen experimentally.

for a breakdown of the OPE.

The precision of this comparison is however strongly limited by the corresponding hadronic parameters. The latest available lattice evaluation of the arising matrix elements stems from 2001⁷⁰. Here clearly an update would be very helpful.

3. The inclusive approach to calculate $\Delta\Gamma_s$ described above, can also be compared to the exclusive approach, where one tries to estimate all exclusive decay channels that contribute to Γ_{12}^s . The seminal paper of Aleksan et. al from 1993⁷¹ was recently updated⁷² and the new results agree well with the predictions using the HQE. Again, an argument in favour of the validity of the HQE.

We have summarized here several theoretical hints for the applicability of the HQE. These hints represent of course not a proof. In the next section we show how the above listed problems of the HQE predictions have been resolved experimentally.

3.2 The missing charm puzzle

The theoretical numbers for n_c are more than ten years old⁷³, so a reanalysis with updated input parameters seems to be desirable. Using a preliminary theoretical result⁷⁴ with input parameters from 2012 and comparing them with the latest value from BaBar in 2006⁷⁵ one finds perfect agreement.

$$n_c^{2006\text{BaBar}} = 1.208_{-0.055}^{+0.058} . \quad (23)$$

$$n_c^{\text{SM}} = 1.20 \pm 0.04 . \quad (24)$$

So there is no hint for a violation of quark hadron duality from the charm counting anymore. Nevertheless it would be very desirable to have updated experimental numbers for n_c from e.g. the full data sets of the B-factories, because this could shrink the allowed region for hypothetical new physics effects contributing to Γ_{12}^q , see below.

3.3 The Λ_b -lifetime

The experimental number³⁴ for the Λ_b -lifetime changed quite dramatically in the last ten years

$$\text{HFAG '03} \quad \tau_{\Lambda_b} = 1.229 \pm 0.080 \text{ ps}^{-1} . \quad (25)$$

$$\text{HFAG '12} \quad \tau_{\Lambda_b} = 1.425 \pm 0.032 \text{ ps}^{-1} . \quad (26)$$

This is a shift of the central value of about 2.5σ . Part of this change is due to the fact that old measurements were only done with semi leptonic Λ_b decays, while for newer measurements also non leptonic decays became available. The newer values can be easily explained by the HQE, e.g.⁷⁶, while the old low values were very problematic. So again, the indications for a failure of the HQE have vanished.

Looking a little closer, one sees however that there are still some issues that have to be settled. Experimentally there seems to be a discrepancy of a about 2σ between lifetime measurements of D0⁷⁷ and CDF⁷⁸ - both measure the non leptonic decay channel. To settle this issue we are eagerly waiting for new LHC results.

But there are also some open theoretical issues, in particular the correct value of the non-perturbative matrix elements of the four-quark operators of the Λ_b -baryon. Due to HQET only two different matrix elements (instead of four) arise. A widely used parameterization is

$$\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \bar{b}_L \gamma_\mu q_L \cdot \bar{q}_L \gamma^\mu b_L | \Lambda_b \rangle =: -\frac{f_B^2 m_B}{48} r . \quad (27)$$

The second matrix element can also be expressed in terms of the parameter r , so in the end $\tau_{\Lambda_b}/\tau_{B_d} - 1$ is proportional to r . Unfortunately we do not have a lattice evaluation of r , only an exploratory study from 1999⁷⁹, whose numerical result should be taken with a lot of caution. In the literature values of r range from 0.2 (QCD sum rules⁸⁰) over 1.2 (the exploratory lattice study⁷⁹) up to 6.2 (QCD sum rules⁸¹ - these results have met however a lot of scepticism in the sum rule community). Rosner related in 1996⁸² the value of r to mass differences of b -hadrons and obtained values between 0.9 and 1.8. Using his results and the recent PDG values⁸³ of the b -hadron masses we get

$$r \approx \frac{4}{3} \frac{m_{\Sigma_b^*}^2 - m_{\Sigma_b}^2}{m_{B^*}^2 - m_B^2} = 0.68 \pm 0.10 , \quad (28)$$

which points towards smaller values of r and consequently to larger values of the Λ_b lifetime. Here clearly new lattice determinations are mandatory to improve the theory prediction.

3.4 The B_s -lifetime

For quite some time the B_s lifetime was also considerably below the B_d lifetime. Just recently several new results for $\Gamma_s = 1/\tau_{B_s}$ were obtained by performing the angular analysis⁸⁴ in the decay $B_s \rightarrow \psi\phi$. LHCb¹⁶, CDF²⁰ and D0⁵ obtained from their fits:

$$\text{LHCb: } \tau_{B_s} = 1.520 \pm 0.020 \text{ ps} . \quad (29)$$

$$\text{CDF: } \tau_{B_s} = 1.528 \pm 0.021 \text{ ps} . \quad (30)$$

$$\text{D0: } \tau_{B_s} = 1.443_{-0.035}^{+0.038} \text{ ps} . \quad (31)$$

Taking the most precise value¹⁶ and the most recent standard model determination¹¹ we find an impressive agreement

$$\frac{\tau_{B_s}^{\text{Exp}}}{\tau_{B_d}} = 1.001 \pm 0.014 , \quad \frac{\tau_{B_s}^{\text{SM}}}{\tau_{B_d}} = 0.996 \dots 1.000 . \quad (32)$$

This again confirms the framework of the HQE. For a more sophisticated comparison of course all lifetime measurements have to be combined properly - this was done also recently by HFAG³⁴ (for this average the most recent LHCb number¹⁶ is not yet included, but the preceding number from¹⁷):

$$\frac{\tau_{B_s}}{\tau_{B_d}} = 0.984 \pm 0.011 . \quad (33)$$

This new average agrees within about 1σ with the very precise theoretical prediction and the HQE seems to work with a high accuracy.

The new lifetime values for the B_s meson can also be used to update the theory predictions for the effective lifetimes. Previously⁸⁵ $\tau_{B_s} = 1.477_{-0.022}^{+0.021}$ ps was used. Replacing this number by the most recent LHCb measurement¹⁶ one finds the new SM model predictions for the effective lifetimes and the flavour-specific lifetime listed below. This can be compared with the experimental numbers for $\tau^{\text{Eff}}(K^+K^-)$ from LHCb⁸⁶, for $\tau^{\text{Eff}}(\psi f_0)$ from CDF⁸⁷ and the average of the flavour specific lifetime from HFAG³⁴:

	Exp.	SM-old	SM-new
$\tau^{\text{Eff}}(K^+K^-)$ in ps	1.468 ± 0.046	1.390 ± 0.032	1.43 ± 0.03
$\tau^{\text{Eff}}(\psi f_0)$ in ps	1.70 ± 0.12	1.582 ± 0.036	1.63 ± 0.03
τ^{FS} in ps	1.463 ± 0.032	— — —	1.54 ± 0.03

For the SM predictions of the effective lifetimes the dominant sources of the error are the theory value of $\Delta\Gamma_s$ (about ± 0.02) and the experimental values for τ_{B_s} . The effective lifetimes agree within 1σ , while the experimental value for the flavour-specific lifetime is about 2.4σ below the theoretical central value. Here also new, more precise data will be very desirable.

3.5 First measurement of $\Delta\Gamma_s$

In Moriond LHCb presented the first measurement ($> 5\sigma$) of a non-zero value of the decay rate difference in the neutral B_s system¹⁶:

$$\Delta\Gamma_s = (0.116 \pm 0.019) \text{ ps}^{-1} . \quad (34)$$

Naively this corresponds to a statistical significance of 6.1σ . $\Delta\Gamma_s$ was obtained from the angular analysis⁸⁴ in the decay $B_s \rightarrow J/\psi\phi$. CDF²⁰ and D0⁵ also performed similar analyses to get:

$$\begin{aligned} \text{D0 } 8\text{fb}^{-1} & \quad \Delta\Gamma_s = (0.163 \pm 0.065) \text{ ps}^{-1} , \\ \text{CDF } 9.6\text{fb}^{-1} & \quad \Delta\Gamma_s = (0.068 \pm 0.026 \pm 0.007) \text{ ps}^{-1} . \end{aligned} \quad (35)$$

The new HFAG average³⁴ reads (for this average the most recent LHCb number¹⁶ is not yet included, but the preceding number from¹⁷):

$$\Delta\Gamma_s = (0.100 \pm 0.013) \text{ ps}^{-1} . \quad (36)$$

There is also a long history of theoretical predictions for $\Delta\Gamma_s$. The first estimates stem from 1977⁵¹ where $\Delta\Gamma_d/\Gamma_d = 1/6$ was found (for B_d mesons!), which is very close to the current value for $\Delta\Gamma_s/\Gamma_s$. As can be seen from Eq.(21) NLO-QCD corrections, subleading HQE corrections and the impact of lattice parameters for $\Delta\Gamma_s$ are ample. Therefore the numerical values of old calculations have to be taken with a pinch of salt, even if some of them are by accident close to the current experimental number.

The subleading $1/m_b$ -corrections were determined in 1996⁵⁷. The dominant NLO-QCD corrections were calculated in 1998⁵⁸, subdominant ones in 2003^{13,14}. In 2006¹² more theoretical improvements were included - now also the relatively well-known bag parameter B_{B_s} gives the dominant contribution. Before that the dominant contribution came from the bag parameter \tilde{B}_S , which describes the matrix elements of the operator \tilde{Q}_S , see. Eq.(16). First lattice estimates of \tilde{B}_S appeared from 2000 on^{88,89,90}. So, before 2000 in principle no reliable estimates of $\Delta\Gamma_s$ could be given at all, because of missing information and before 2006 estimates were affected by large uncertainties. More recent evaluations^{12,11} readⁱ

$$2006 : \quad \Delta\Gamma_s = (0.096 \pm 0.036) \text{ ps}^{-1} , \quad (37)$$

$$2011 : \quad \Delta\Gamma_s = (0.087 \pm 0.021) \text{ ps}^{-1} . \quad (38)$$

We will stick to Eq.(38) as our theory prediction for $\Delta\Gamma_s$. Comparing experiment with theory we again find a perfect agreement:

$$\frac{\Delta\Gamma_s^{\text{Exp.}}}{\Delta\Gamma_s^{\text{SM}}} = \frac{0.100 \pm 0.013}{0.087 \pm 0.021} = 1.15 \pm 0.32 . \quad (39)$$

This result proves experimentally that the HQE works also for the calculation of Γ_{12}^q . The issue of the proper calculation of Γ_{12}^q gave raise to numerous discussions in the literature, see e.g. ⁶² and references and citations of this paper. This issue is now settled.

Now several comments to Eq.(39) are appropriate:

ⁱThe Rome Group will soon present⁹¹ a numerical update, which yields $\Delta\Gamma_s/\Gamma_s = 0.149 \pm 0.015$. This corresponds to $\Delta\Gamma_s = 0.098 \pm 0.010 \text{ ps}^{-1}$ and is in perfect agreement with the measurement. It also agrees with the result from¹¹.

- Eq.(39) shows that the HQE works for Γ_{12}^q with an accuracy of $15\% \pm 32\%$. Therefore the question is not anymore, whether the HQE is appropriate for Γ_{12}^q , but how precise is the HQE. Does it work to an accuracy of 30%? Or maybe even to an accuracy of 10%? This can be answered by further reducing the theoretical and the experimental error.
- The uncertainty in Eq.(39) is dominated by the theoretical error, but an improvement of the experimental error will also be helpful. Here new data from LHCb and the two planned Super-B-factories^{92,93,94} will be very important.
- The theory prediction for Γ_{12}^q has again a crucial dependence on non-perturbative parameters. In particular it is also proportional to $f_{B_s}^2$. For the theory prediction we have used again $f_{B_s} = 231 \pm 15$ MeV. As in the case of M_{12}^q we also show the results, if new evaluations of the decay constant are used^{41,42}.

Reference	f_{B_s}	$\Delta\Gamma_s$
HPQCD (1110.4510)	225 ± 4 MeV	0.083 ± 0.017 ps ⁻¹
Fermilab/MILC (1112.3051)	242 ± 10 MeV	0.096 ± 0.021 ps ⁻¹

(40)

In contrast to M_{12}^q now the error is not reduced so strongly, if one uses smaller errors for the decays constant. For the same reasons as discussed in relation to ΔM_s we will use the prediction from Eq.(38).

- To see, what is necessary to increase the theoretical accuracy, we compare the error budget from the theory prediction of $\Delta\Gamma_s$ in 2006¹² with the one from 2011¹¹:

$\Delta\Gamma_s^{\text{SM}}$	2011	2006
Central Value	0.087 ps ⁻¹	0.096 ps ⁻¹
$\delta(\mathcal{B}_{\tilde{R}_2})$	17.2%	15.7%
$\delta(f_{B_s})$	13.2%	33.4%
$\delta(\mu)$	7.8%	13.7%
$\delta(\mathcal{B}_{S,B_s})$	4.8%	3.1%
$\delta(\mathcal{B}_{R_0})$	3.4%	3.0%
$\delta(V_{cb})$	3.4%	4.9%
$\delta(\mathcal{B}_{B_s})$	2.7%	6.6%
...
$\sum \delta$	24.5%	40.5%

Currently the dominant theoretical uncertainty stems from the power suppressed operator R_2 . These matrix elements have currently only been estimated with vacuum insertion approximation. Numerical subleading effects within the QCD sum rule approach have been determined in^{95,96}. Here a more precise determination of B_{R_2} is mandatory in order to improve the theoretical accuracy of Γ_{12}^q .

- For a long time the dominant error source of the theory prediction of $\Delta\Gamma_s$ was f_{B_s} , currently the dependence on $f_{B_s}^2$ is the second largest error. Thus one might want to get rid off the dependence on f_{B_s} by considering the ratio $\Delta\Gamma_s/\Delta M_s$ - here one implicitly also assumes that no new physics acts in ΔM_s . In this ratio the overall factor $f_{B_s}^2$ cancels, as well as the bag parameter B_{B_s} of the operator Q , given in Eq.(8):

$$\begin{aligned}
\frac{\Delta\Gamma_s}{\Delta M_s} &= 10^{-4} \cdot \left[46.2 + 10.6 \frac{\tilde{B}'_S}{B} - \left(13.2 \frac{B_{\tilde{R}_2}}{B} - 2.5 \frac{B_{R_0}}{B} + 1.2 \frac{B_R}{B} \right) \right] \\
&= 0.0050 \pm 0.0010 .
\end{aligned}
\tag{41}$$

The numerical dominant term is now completely free of hadronic uncertainties, the remaining dependence on hadronic parameters is always via ratios of bag parameters. Comparing this theory prediction¹¹ with the experimental number presented by HFAG³⁴ one gets

$$\left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{Exp}} \bigg/ \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right)^{\text{SM}} = 1.12 \pm 0.27. \quad (42)$$

Once more an impressive agreement with the HQE prediction and the assumption that no new physics acts in ΔM_s or $\Delta\Gamma_s$.

3.6 $\Delta\Gamma_s$ from $Br(B_s \rightarrow D_s^{(*)+} + D_s^{(*)-})$?

In the literature one finds regularly the following relation

$$\frac{\Delta\Gamma_s^{\text{CP}}}{\Gamma_s} = 2Br(B_s \rightarrow D_s^{(*)+} + D_s^{(*)-}) \quad (43)$$

and one might of course be tempted to use Eq.(43), to determine $\Delta\Gamma_s^{\text{CP}} = |\Gamma_{12}^s|$ directly from “simple” branching ratio measurements. This equation was derived in 1993⁷¹ to show the equivalence of the exclusive and inclusive approach for calculating $\Delta\Gamma_s$. It holds in the limit $m_c \rightarrow \infty; m_b - 2m_c \rightarrow 0; N_c \rightarrow \infty$. This limit corresponds to neglecting the 3-body final state contributions to Γ_{12}^s . In 1993 Aleksan et al. estimated⁷¹

$$\frac{\Delta\Gamma_s}{\Gamma_s} \propto \mathcal{O}(0.15), \quad (44)$$

which is amazingly close to the current experimental value³⁴. This analysis was redone recently⁷². With up to date values of the input parameters one gets: the 2-body final states contribute 0.100 ± 0.030 to $\Delta\Gamma_s/\Gamma_s$. This number is the updated value of Eq.(44). It is slightly below the HQE prediction in Eq.(38). Moreover it turned out that the 3-body final state contributions - which are neglected in the derivation of Eq.(43) - are quite sizable, they contribute about 0.06...0.08 to $\Delta\Gamma_s/\Gamma_s$. Since this is comparable to the 2-body final states, the above approximation turns out to be a bad one. We repeat here the suggestion from¹²:

We strongly discourage from the inclusion of $Br(B_s \rightarrow D^{()+} + D^{(*)-})$ in averages with $\Delta\Gamma_s$ determined from clean methods.*

3.7 HQE at work: semi leptonic CP asymmetries and mixing phases

In the previous sections we have shown that the HQE describes inclusive decay of b-hadrons very accurate. Even the prediction of Γ_{12}^s agrees with an accuracy of about 30% with the experiment. To make more precise statements, some non-perturbative progress is mandatory.

Because of this success we use now the HQE to predict¹¹ also quantities that might be very sensitive to new physics effects like semi leptonic CP asymmetries and mixing phases.

$$a_{fs}^s = (1.9 \pm 0.3) \cdot 10^{-5}, \quad \phi_s = 0.22^\circ \pm 0.06^\circ, \quad (45)$$

$$a_{fs}^d = -(4.1 \pm 0.6) \cdot 10^{-4}, \quad \phi_d = -4.3^\circ \pm 1.4^\circ, \quad (46)$$

$$A_{sl}^b = 0.406a_{sl}^s + 0.594a_{sl}^d = (-2.3 \pm 0.4) \cdot 10^{-4}. \quad (47)$$

The semi leptonic CP asymmetries turn out to be extremely small, as well as the mixing phase in the B_s -system. Therefore any measurement of a sizable value would be a clear indication of new physics. The so-called di-muon asymmetry A_{sl}^b is a linear combination of the semi-leptonic asymmetries in the B_d and B_s systems. As mentioned in the the introduction there was a

considerable excitement in the past, because we had several hints for huge new physics effects in B -mixing. Before the Lepton-Photon conference 2011 a fit¹ of flavour observables resulted in a huge deviation of the B_s mixing phase from the tiny SM prediction:

$$\phi_s = \phi_s^{\text{SM}} + \phi_s^\Delta, \quad (48)$$

$$\phi_s^\Delta = -51.6^\circ \pm 12^\circ, \quad (49)$$

where ϕ_s^Δ denotes the deviation from the SM value of the B_s -mixing phase. Moreover the D0 collaboration measured^{6,7,8} a large deviation of the di-muon asymmetry from the theory expectation:

$$A_{sl}^b = -(7.87 \pm 1.72 \pm 0.93) \cdot 10^{-3}. \quad (50)$$

This number is by a factor of 34 larger than the SM expectation, the statistical significance of the deviation is 3.9σ . The di-muon asymmetry currently still has its large value, while the hints for large deviations of ϕ_s from zero have disappeared. A fit of the most recent data, e.g.¹⁶ from $B_s \rightarrow J/\psi\phi$ gives

$$\phi_s^\Delta = -0.1^\circ \pm 6.1^\circ. \quad (51)$$

This number is perfectly consistent with the SM, while still some ample deviations are allowed. Current experimental bounds on the individual semi leptonic CP asymmetries are given by HFAG³⁴

$$a_{fs}^s = (-1050 \pm 640) \cdot 10^{-5}, \quad (52)$$

$$a_{fs}^d = -(33 \pm 33) \cdot 10^{-4}. \quad (53)$$

These remaining deviations will be investigated systematically below.

3.8 The decay rate difference in the B_d -system

The SM predicts¹¹ also a tiny width difference in the B_d system.

$$\frac{\Delta\Gamma_d}{\Gamma_d} = (4.2 \pm 0.8) \cdot 10^{-3}. \quad (54)$$

This can be compared to the current most precise bound³² from Belle.

$$\frac{\Delta\Gamma_d}{\Gamma_d} = (-17 \pm 21) \cdot 10^{-3}. \quad (55)$$

$\Delta\Gamma_s$ is governed by the CKM-leading tree-level decay $b \rightarrow c\bar{c}s$. So any sizable new physics effect to this decays would also affect strongly many other flavour observables. $\Delta\Gamma_d$ is governed by Cabibbo-suppressed decays, hence the bounds on new physics effects in $\Delta\Gamma_d$ are weaker and the real value of Δ_d might deviate from its almost vanishing SM value. This interesting null-test of the SM was discussed recently in⁹⁷.

4 New Physics in B -mixing

New physics in B -mixing can be parameterized model independently in the following way

$$\Gamma_{12}^q = \Gamma_{12}^{q,\text{SM}}, \quad M_{12}^q = M_{12}^{q,\text{SM}} \cdot \Delta_q, \quad \Delta_q = |\Delta_q| e^{i\phi_q^\Delta}. \quad (56)$$

Here we assume that new physics acts only in M_{12}^q , while Γ_{12}^q is given by the SM prediction. Although this is expected to hold within the hadronic uncertainties (up to 25%), we will also

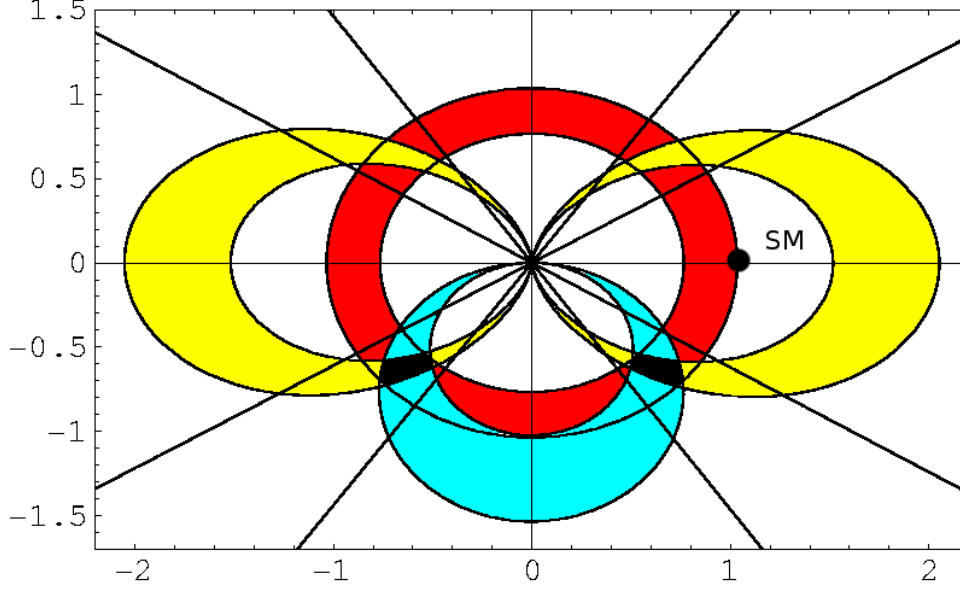


Figure 2: Bounds on the complex Δ_q -plane from ΔM_q (red), $\Delta\Gamma_q/\Delta M_q$ (yellow), a_{sl}^q (turquoise) and direct determinations of ϕ_q (black rays).

investigate new physics effects in Γ_{12}^q in the next section.

Using the parameterization of Eq.(56) one can express the observables in the mixing system in terms of the SM predictions for M_{12}^q and Γ_{12}^q and in terms of the complex parameter Δ_q :

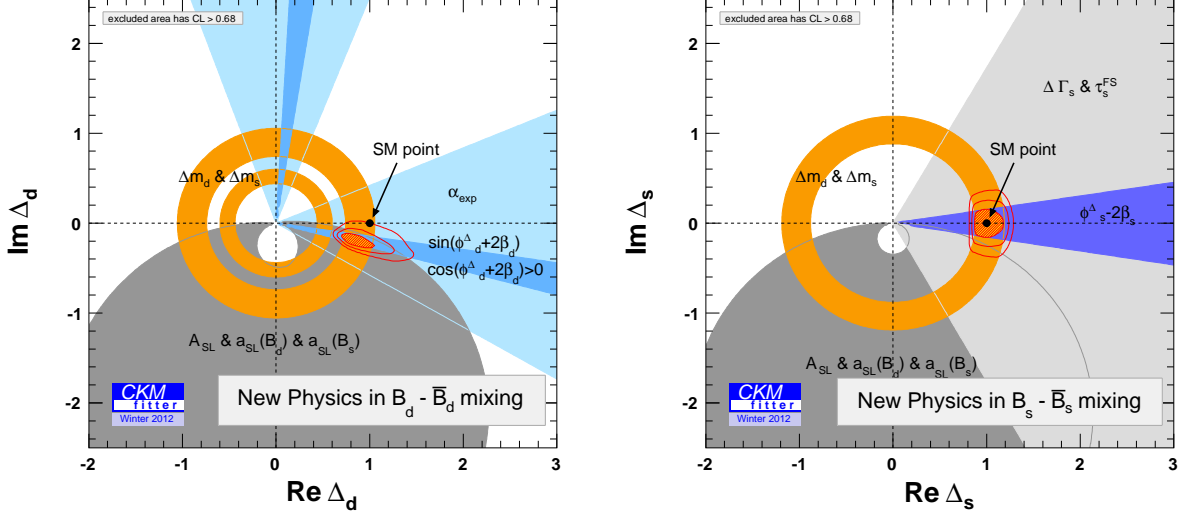
$$\Delta M_q = 2|M_{12}^{q,\text{SM}}| \cdot |\Delta_q|, \quad (57)$$

$$\Delta\Gamma_q = 2|\Gamma_{12}^q| \cdot \cos(\phi_q^{\text{SM}} + \phi_q^\Delta), \quad (58)$$

$$\frac{\Delta\Gamma_q}{\Delta M_q} = \frac{|\Gamma_{12}^q|}{|M_{12}^{q,\text{SM}}|} \cdot \frac{\cos(\phi_q^{\text{SM}} + \phi_q^\Delta)}{|\Delta_q|}, \quad (59)$$

$$a_{fs}^q = \frac{|\Gamma_{12}^q|}{|M_{12}^{q,\text{SM}}|} \cdot \frac{\sin(\phi_q^{\text{SM}} + \phi_q^\Delta)}{|\Delta_q|}. \quad (60)$$

Noticing $\sin(\phi_s^{\text{SM}}) \approx 1/240$, one sees that new physics easily could enhance the semi leptonic asymmetries in the B_s -system by a factor of almost 250, if a sizable phase ϕ_s^Δ is present in M_{12}^s . The relations of Eq.(57) - Eq.(60) can be used to obtain bounds on the complex Δ_q -plane. As inputs one needs the SM predictions for M_{12}^q and Γ_{12}^q , as well as the experimental numbers for ΔM_q , $\Delta\Gamma_q$, a_{sl}^q and $\phi_q = \phi_q^{\text{SM}} + \phi_q^\Delta$. For the hypothetical case $|\Delta_q| = 0.9$ and $\phi_q^\Delta = -\pi/4$ one gets the bounds of Fig.(2) in the complex Δ_q -plane. Eq.(57) gives a bound on the modulus of Δ_q , this corresponds to a circle in the complex Δ_q -plane (red region in Fig.(2)). Eq.(58) gives a direct bound on the phase, this corresponds to the black rays in Fig.(2). Because of the relatively large theory uncertainties in Γ_{12}^q , it is however much more advantageous to use a direct determination of ϕ_q , e.g. for ϕ_s from the angular analysis of decays like $B_s \rightarrow J/\psi\phi$ or $B_s \rightarrow J/\psi f_0$ ⁹⁸. Eq.(59) and Eq.(60) constrain simultaneously the modulus of Δ_q and the phase ϕ_q^Δ . $\Delta\Gamma_q/\Delta M_q$ gives the yellow region in Fig.(2), while a_{sl}^q gives the turquoise one. Combining all data till march 2012 and allowing only for new physics in B_d - and B_s -mixing (i.e. neglecting e.g. new physics penguin contributions), one gets the following bounds on the complex Δ_d - and Δ_s -planes¹⁹ (update of¹).



In 2010¹ new physics in B -mixing could very well accommodate the different deviations from the SM expectations, seen at that time. This is not the case anymore in 2012¹⁹. There is now a tension between the direct determination of ϕ_s and the di-muon asymmetry.

In the B_d -system, new physics in $M_{12,d}$ can resolve the discrepancy between $B \rightarrow \tau\nu$ and direct determinations of $\sin 2\beta$. In the B_s -system everything looks SM-like although still sizable values for ϕ_s^Δ are possible. Just recently a second (symmetric) solution in the complex Δ_s -plane was excluded⁹⁹. We also would like to note that in¹⁹ no tension is found for ϵ_K .

To improve further the bounds on the complex Δ_q -planes, more precise data are necessary.

5 New Physics in Γ_{12}

The theory expression for the di-muon asymmetry can be written in the following way

$$A_{sl} = (0.594 \pm 0.022)(5.4 \pm 1.0) \cdot 10^{-3} \frac{\sin(\phi_d^{SM} + \phi_d^\Delta)}{|\Delta_d|} + (0.406 \pm 0.022)(5.0 \pm 1.1) \cdot 10^{-3} \frac{\sin(\phi_s^{SM} + \phi_s^\Delta)}{|\Delta_s|}. \quad (61)$$

Since Δ_s and Δ_d are bounded from measurements of the mass differences to be close to one and the sine can be at most one, there exists a theoretical upper limit for the di-muon asymmetry. We use here the fit values of Δ_q from¹⁹ to obtain the following upper bounds:

$$A_{sl} \leq \begin{cases} -1.7 \cdot 10^{-3} : & 1\sigma \text{ for } |\Delta_q|, \quad 1\sigma \text{ for } \phi_q^\Delta, \\ -2.8 \cdot 10^{-3} : & 3\sigma \text{ for } |\Delta_q|, \quad 3\sigma \text{ for } \phi_q^\Delta, \\ -7.5 \cdot 10^{-3} : & 3\sigma \text{ for } |\Delta_q|, \quad \text{set sine to 1.} \end{cases} \quad (62)$$

For the first number the four parameters of Δ_q ($q=s,d$) have been chosen to take the value, which gives the largest di-muon asymmetry, within the allowed 1σ range of the fit in¹⁹, for the second number, the 3σ range has been chosen, while for the third number the sine has been set to one by hand. The last number is purely hypothetical, because such a large value of the mixing phase is in contrast to experimental investigations of e.g. $B_s \rightarrow J/\psi\phi^j$. The above

^jThis also holds, if one takes into account large new physics penguin contributions to the decay $b \rightarrow c\bar{c}s$, which could lead to a certain extent to a cancellation between the penguin phase and ϕ_q^Δ . See the discussion in the next section.

bounds have to be compared with the experimental measurement^{6,7,8}

$$A_{sl}^{D0} = (-7.8 \pm 2.0) \cdot 10^{-3}. \quad (63)$$

One finds that the central value of the di-muon asymmetry is larger than *theoretically* possible, see also¹⁰⁰. This discrepancy triggered a lot of interest in the literature (see the list of citations for^{6,7,8}). The theoretical upper limit stems from the fact that we did not allow ample (i.e. larger than the hadronic uncertainties, which are of the order of 25 %) deviations of Γ_{12}^q from its SM value. One might of course be tempted to give up this assumption. If there is no new physics acting in M_{12}^q one would need an enhancement factor for Γ_{12}^q of 34 to describe the central value of the di-muon asymmetry, if new physics acts in M_{12}^q one still needs an enhancement factor of about 4.6 (1 σ -range of Eq.(62)) or 2.8 (3 σ -range of Eq.(62)).

As discussed above violations of quark hadron duality in the range of 280% – 3400% are clearly ruled out now, as experiment and theory agree for $\Delta\Gamma_s$ to an accuracy of about 30%.

Next one might think of new physics effects in Γ_{12}^q . One can modify Eq.(56) to allow for new physics in Γ_{12}^q :

$$\begin{aligned} \Gamma_{12}^q &= \Gamma_{12}^{q,\text{SM}} \cdot \tilde{\Delta}_q, & M_{12}^q &= M_{12}^{q,\text{SM}} \cdot \Delta_q, \\ \tilde{\Delta}_q &= |\tilde{\Delta}_q| e^{i\phi^{\tilde{\Delta}_q}}, & \Delta_q &= |\Delta_q| e^{i\phi_q^\Delta}. \end{aligned} \quad (64)$$

Just looking at ΔM_q , $\Delta\Gamma_q$ and a_{sl}^q one can of course fit any numerical value of these three observables with the three parameters $|\Delta_q|$, $|\delta_q|$ and $\phi_q^\Delta - \phi_q^\delta$.

$$\Delta M_q = 2|M_{12}^{q,\text{SM}}| \cdot |\Delta_q|, \quad (65)$$

$$\Delta\Gamma_q = 2|\Gamma_{12}^{q,\text{SM}}| \cdot |\tilde{\Delta}_q| \cdot \cos(\phi_q^{\text{SM}} + \phi_q^\Delta - \phi^{\tilde{\Delta}_q}) , \quad (66)$$

$$a_{fs}^q = \frac{|\Gamma_{12}^{q,\text{SM}}|}{|M_{12}^{q,\text{SM}}|} \cdot \frac{|\tilde{\Delta}_q|}{|\Delta_q|} \cdot \sin(\phi_q^{\text{SM}} + \phi_q^\Delta - \phi^{\tilde{\Delta}_q}) . \quad (67)$$

So the situation $\Delta M_q = \Delta M_q^{\text{SM}}$, $\Delta\Gamma_q = \Delta\Gamma_q^{\text{SM}}$ and $a_{sl}^q = 34a_{sl}^{q,\text{SM}}$ can be described by $|\Delta_q| = 1$, $|\tilde{\Delta}_q| \cdot \cos(\phi_q^{\text{SM}} + \phi_q^\Delta - \phi^{\tilde{\Delta}_q}) = 1$ and $|\tilde{\Delta}_q|/|\Delta_q| \cdot \sin(\phi_q^{\text{SM}} + \phi_q^\Delta - \phi^{\tilde{\Delta}_q}) = 1$, which is equivalent to $|\Delta| = 1$, $|\tilde{\Delta}| = 34.0147$ and $\cos(\phi_q^{\text{SM}} + \phi_q^\Delta - \phi^{\tilde{\Delta}_q}) = 0.0293991$. In this spirit bounds on new physics contributions to Γ_{12}^d and Γ_{12}^s were obtained in¹⁹, see Fig.(3). It turned out to be useful to introduce a new parameter δ_q :

$$\delta_q = \frac{\frac{\Gamma_{12}^q}{M_{12}^q}}{\text{Re}\left(\frac{\Gamma_{12}^{q,\text{SM}}}{M_{12}^{q,\text{SM}}}\right)}, \quad (68)$$

$$\text{Re}(\delta_q) = \frac{\Delta\Gamma_q}{\Delta M_q} \bigg/ \left(\frac{\Delta\Gamma_q^{\text{SM}}}{\Delta M_q^{\text{SM}}} \right), \quad (69)$$

$$\text{Im}(\delta_q) = -a_{sl}^q \bigg/ \left(\frac{\Delta\Gamma_q^{\text{SM}}}{\Delta M_q^{\text{SM}}} \right). \quad (70)$$

The experimental constraint on $\Delta\Gamma_s$ can be fulfilled by $\text{Re } \delta_s \approx 1$ and the semi leptonic asymmetries only affect $\text{Im } \delta_s$. The real part of δ_d is almost unconstrained: $\text{Re } \delta_d \approx -4.3 \pm 5.4$ (see Section 3.8), while the the semi leptonic asymmetries again affect only the imaginary part of δ_d . Γ_{12}^s is dominated by the Cabibbo-favoured decay $b \rightarrow c\bar{c}s$. Thus any sizable contribution to Γ_{12}^s will affect many other observables dramatically. As explained in¹⁹ these effects on other observables have to be taken into account, when using the results of Fig(3)! Any new physics

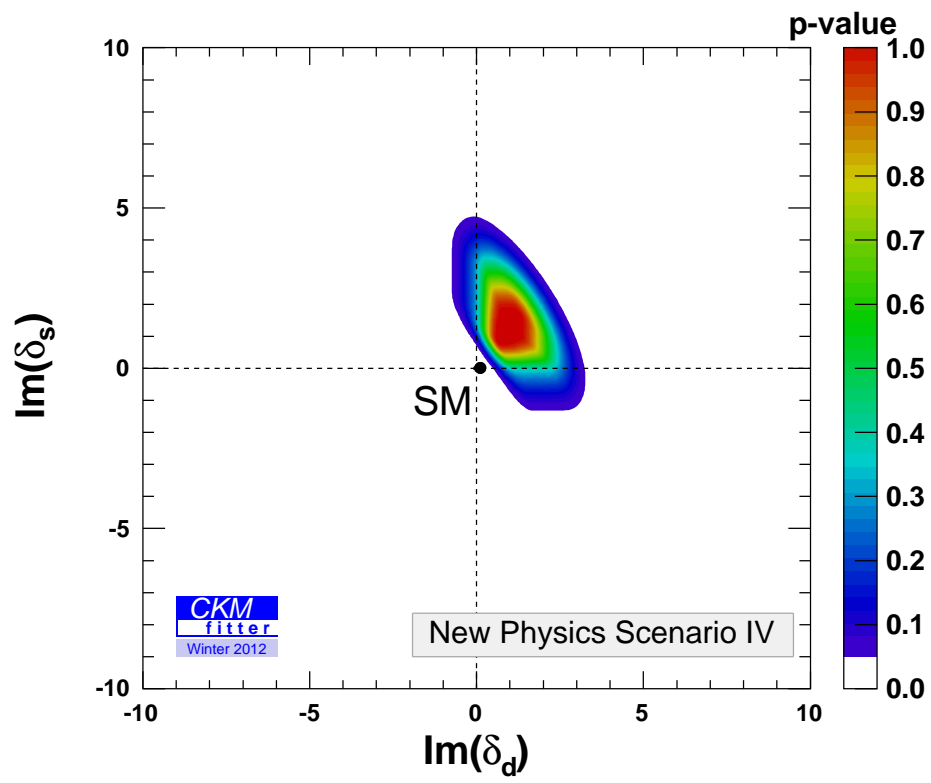
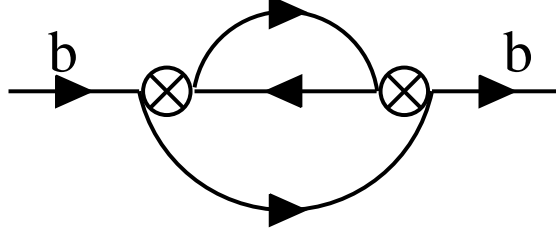
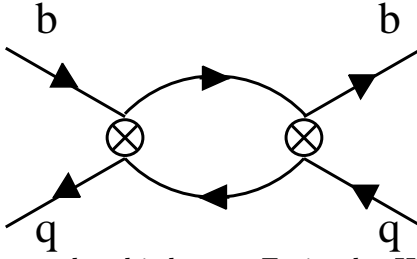


Figure 3: Bounds on the parameter $\text{Im}(\delta_q)$, if only ΔM_q , $\Delta \Gamma_q$ and a_{sl}^q are taken into account. Large values of $\text{Im}(\delta_s)$ violate however the perfect agreement of experiment and theory for quantities like $\tau_{B_s}/\tau_{B_d}, \dots$ (see text).

contribution to Γ_{12}^s corresponds to a transition $b\bar{s} \rightarrow X$, where X can be one or more (SM or new physics) particles and $M_X \leq M_{B_s}$. The new operator $(b\bar{s}X)$ gives now rise to many different new contributions. Via diagrams like (if X corresponds to two particles)



the $(b\bar{s}X)$ -operator contributes to the leading term Γ_0 in the HQE, see. Eq.(20). This affects the total decay rate Γ_s and therefore quantities like the semi leptonic branching ratio, or the branching ratio of a b -quark into two, one or zero charm quarks (see the discussion of the missing charm puzzle in Section(3.2)). Via diagrams like



the $(b\bar{s}X)$ -operator contributes to the third term Γ_3 in the HQE, see. Eq.(20). This affects the lifetime ratio $\tau(B_s)/\tau(B_d)$, but also the decay rate difference $\Delta\Gamma_s$ and the semi leptonic CP-asymmetries^k. It will also give rise to new contributions to M_{12}^s . Via mixing such a new operator can also modify well constrained quantities like the rare decay $b \rightarrow s\gamma$.

All in all, any new physics contribution to Γ_{12}^s will face severe constraints from many well measured quantities. Hence it seems impossible that ample (i.e. larger than the hadronic uncertainties, which are of the order of 25%) new physics effects are present in Γ_{12}^s . Unfortunately this statement cannot be proven model independently. At least the operator structure of the new contribution has to be specified, so that one can also calculate the new contribution to e.g. Γ_0 and Γ_3 . Such an analysis⁶⁸ has been done for the most promising (because least constrained) candidate, $X = \tau^+ + \tau^-$ and it turned out that new physics contributions to Γ_{12}^s have to be smaller than 30% or 40%, depending on the Dirac-structure of the $bs\tau\tau$ -operator. This clearly excludes the possibility of having new physics contributions to Γ_{12}^s in the range of 280%–3400%. More precise data on τ_{B_s}/τ_{B_d} , B_{sl} , n_c , $Br(B_s \rightarrow \tau^+\tau^-)$,... will further shrink the bound on new physics effects in Γ_{12}^s (or give hints for small new physics effects).

Finally there is also the possibility that D0 simply saw a 2.5 σ upward fluctuation of the result for the di-muon asymmetry and that the actual value is below -2.8 per mille, which still would be a large deviation from the SM. To get a final clue about that, independent measurements of the semi leptonic asymmetries are urgently needed.

6 Penguin pollution

In the last section I will discuss the question to what extent does the value of the di-muon asymmetry contradict the result of analyses of $B_s \rightarrow J/\psi\phi$ from LHCb and Tevatron. The angular analysis of the decay $B_s \rightarrow J/\psi\phi$ at CDF, D0 and LHCb gives among others the quantity $S_{\psi\phi}$. If one neglects certain penguin contributions, then $S_{\psi\phi}$ is given in the SM simply

^kStrictly speaking the contributions to Γ_{12}^q and Γ_3 are of course not equal, see e.g. the discussion in ⁶³.

by CKM-elements. Its SM expectation reads¹¹

$$S_{\psi\phi}^{\text{SM}} = 0.036 \pm 0.002 . \quad (71)$$

In the presence of new physics $S_{\psi\phi}^{\text{SM}}$ will be modified by the new mixing phase ϕ_s^Δ , which also changes the values of the semi leptonic CP-asymmetries

$$S_{\psi\phi}^{\text{SM}} \rightarrow \sin(2\beta_s - \phi_s^\Delta) = 0.00 \pm 0.11 . \quad (72)$$

As already mentioned above, in deriving e.g. Eq.(72) certain penguin contributions have been neglected. Since the current experimental bound on $S_{\psi\phi}$ (taken from¹⁹) has already a small error, this approximation might not be justified any more. It was already pointed out in¹⁰¹ that a measurement of a relatively small value of the mixing phase requires a much more stringent control of the penguin contributions. Including penguins one gets the general relation

$$S_{\psi\phi}^{\text{SM}} \rightarrow \sin(2\beta_s - \phi_s^\Delta - \delta_s^{\text{Peng,SM}} - \delta_s^{\text{Peng,NP}}) . \quad (73)$$

Looking at Eq.(73) one sees that there is the possibility that a non-vanishing value of the new mixing phase ϕ_s^Δ is compensated by SM and new physics penguins. Now it depends on the possible size of the penguins, to what extent such a cancellation can occur.

To exactly determine the size of the penguin contributions one would have to solve all the non-perturbative dynamics related to this decay, which is clearly not possible. For a rough numerical estimate we start with the amplitude

$$A(B_s \rightarrow f) = \langle B_s | \mathcal{H}_{eff} | f \rangle , \quad (74)$$

with the effective SM hamiltonian for $b \rightarrow c\bar{c}s$ transitions

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left[\lambda_u (C_1 Q_1^u + C_2 Q_2^u) + \lambda_c (C_1 Q_1^c + C_2 Q_2^c) + \lambda_t \sum_{i=3}^6 C_i Q_i \right] . \quad (75)$$

The CKM structure is given by $\lambda_x := V_{xb}^* V_{xs}$; the decay $b \rightarrow c\bar{c}s$ proceeds via the current-current operators Q_1^c, Q_2^c and the QCD penguin operators Q_3, \dots, Q_6 . C_1, \dots, C_6 are the corresponding Wilson coefficients. When the current-current operators Q_1^u, Q_2^u are inserted in a penguin diagram in the effective theory, they also contribute to $b \rightarrow c\bar{c}s$. Electro-weak penguins are neglected.

Therefore we have the following structure of the amplitude $A(B_s \rightarrow f)$

$$A = \frac{G_F}{\sqrt{2}} \left[\lambda_u \sum_{i=1,2} C_i \langle Q_i^u \rangle^P + \lambda_c \sum_{i=1,2} C_i \langle Q_i^c \rangle^{T+P} + \lambda_t \sum_{i=3}^6 C_i \langle Q_i^u \rangle^T \right] . \quad (76)$$

$\langle Q \rangle^T$ denotes the tree-level insertion of the local operator Q , $\langle Q \rangle^P$ denotes the insertion of the operator Q in a penguin diagram. Using unitarity $\lambda_t = -\lambda_u - \lambda_c$ we can rewrite the amplitude in the following form.

$$A = \frac{G_F}{\sqrt{2}} \lambda_c \left[\sum_{i=1,2} C_i \langle Q_i^c \rangle^{T+P} - \sum_{i=3}^6 C_i \langle Q_i^u \rangle^T + \frac{\lambda_u}{\lambda_c} \left(\sum_{i=1,2} C_i \langle Q_i^u \rangle^P - \sum_{i=3}^6 C_i \langle Q_i^u \rangle^T \right) \right] . \quad (77)$$

The first term gives rise to β_s in the CP asymmetry. The second term (proportional to λ_u), corresponds to the SM penguin pollution. To get an idea about the possible size of the second term we notice⁴³ that $\text{Re}(\lambda_u/\lambda_c) < 0.01$ and $\text{Im}(\lambda_u/\lambda_c) < 0.02$. Moreover the penguin Wilson coefficients $|C_{3,\dots,6}|$ are typically smaller than 0.04, therefore one can neglect them in

comparison to the Wilson coefficients $C_1, C_2 \approx 1$. For inclusive decays it turned out that 73 $\langle Q \rangle^P \leq 0.05 \langle Q \rangle^T$, which would result in a penguin pollution smaller than $0.0005 + 0.0010i$, i.e. smaller than one per mille compared to the contribution of the golden plate mode and thus completely negligible, even if one takes into account the current experimental precision. Of course this estimate is very naive and it might change considerably if the matrix elements of the exclusive final states are taken into account, but also a non-perturbative enhancement of this estimate by a factor of 10 would not change the conclusion.

Here clearly more theoretical work has to be done to quantify the size of penguin contributions, Experimental strategies to determine the penguin contributions to $B_s \rightarrow \psi\phi$ and $B_s \rightarrow \psi f_0$ have been discussed in e.g. 101 and the second reference of 85 . An investigation of the decay $B_s \rightarrow \psi K_S$ at the LHCb upgrade can also gain important insights on the size of penguin contributions 102 . Nevertheless it seems that the penguin pollution in the SM is tiny, except some unknown non-perturbative would arise.

In principle new physics penguins might be much larger, but it is hard to imagine that a phase ϕ_s^Δ of order one can be compensated by penguins, without violation other constraints. Therefore the angular analysis of $B_s \rightarrow \psi\phi$ seems to be in a slight contraction to the central value of the di-muon asymmetry.

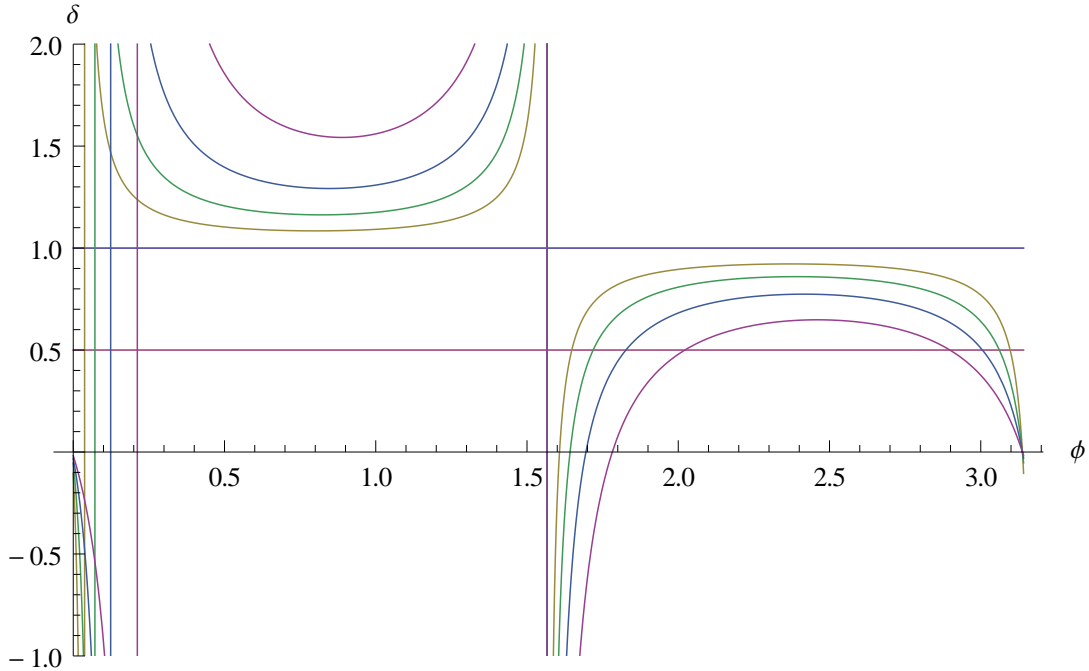
Finally we will show that even small penguin contributions have an observable effect and that they also can be analysed quite model independent 69 . The mass difference, the decay rate difference, the semi leptonic CP asymmetries and $S_{\psi\phi}$ are related via

$$a_{sl}^s = -\frac{\Delta\Gamma}{\Delta M} \frac{S_{\psi\phi}}{\sqrt{1 - S_{\psi\phi}^2}} \cdot \delta, \quad (78)$$

with

$$\delta = \frac{\tan(\phi_s^{\text{SM}} + \phi_s^\Delta)}{\tan(-2\beta_s^{\text{SM}} + \phi_s^\Delta + \delta_s^{\text{peng,SM}} + \delta_s^{\text{peng,NP}})}. \quad (79)$$

All SM phases are small: $\phi_s^{\text{SM}} = 0.22^\circ \pm 0.06^\circ$ and $-2\beta_s = (2.1 \pm 0.1)^\circ$. Nevertheless they are non-negligible 69 . This can be seen by drawing δ in dependence of ϕ^Δ for different values of the penguin contributions, $\delta_s^{\text{peng,SM}} + \delta_s^{\text{peng,NP}} = 0^\circ, 2^\circ, 5^\circ, 10^\circ$.



The curve closest to $\delta = 1$ corresponds to $\delta_s^{\text{peng,SM}} + \delta_s^{\text{peng,NP}} = 0^\circ$, the next one to 2° and so on. Because of this huge sensitivity to penguin effects Eq.(78) can be used to gain some insight in the size of penguin contributions.

7 Conclusion

In Moriond 2012 the first measurement of $\Delta\Gamma_s$ was presented from LHCb. Combining the LHCb data with the corresponding ones from CDF and D0 and comparing their average with theory predictions one finds an impressive agreement:

$$\frac{\Delta\Gamma_s^{\text{Exp.}}}{\Delta\Gamma_s^{\text{SM}}} = \frac{0.100 \pm 0.013}{0.087 \pm 0.021} = 1.15 \pm 0.32 . \quad (80)$$

This represents the first experimental proof that the HQE can also be applied to the calculation of Γ_{12}^s , with an uncertainty of about 30%, which is an important theoretical insight, because Γ_{12}^s is expected to be most sensitive to violations of quark hadron duality. To some extent this is quite amazing, because the energy release in the dominant decays, contributing to $\Delta\Gamma_s$ is not so large: $m_{B_s} - 2m_{D_s} \approx 1.428$ GeV. Currently the experimental and the theoretical errors for $\Delta\Gamma_s$ are of similar size. To improve the theoretical accuracy, matrix elements of higher dimensional operators have to be determined non-perturbatively.

Total decay rates, i.e. lifetimes, are expected to be less sensitive to violations of quark-hadron duality. There are now several precise numbers for $\tau(B_s)$ available from CDF and LHCb, which also agree perfectly with theory predictions.

$$\frac{\tau_{B_s}^{\text{Exp}}}{\tau_{B_d}} = 1.001 \pm 0.014 , \quad \frac{\tau_{B_s}^{\text{SM}}}{\tau_{B_d}} = 0.996 \dots 1.000 . \quad (81)$$

This is again a perfect confirmation of the applicability of HQE. In the case of the lifetime ratios τ_{B^+}/τ_{B_d} and $\tau_{\Lambda_b}/\tau_{B_d}$ no deviation is currently existing, the precision of any comparison is however strongly limited by our poor knowledge of the corresponding hadronic parameters and also by the uncertain experimental value of τ_{Λ_b} .

In accordance with this perfect agreement between experiment and theory, the new LHC and Tevatron data show no further hints for new physics in B_s -mixing. A model independent fit of all flavour data is consistent with no new physics in B_s -mixing (although there is still some room for sizable deviations from the SM expectations) and some small deviations in B_d -mixing. To investigate these issues further a better control over penguin contributions is mandatory.

The large central value of the di-muon asymmetry is still an unsolved problem, because it can not be explained by new physics contributions to M_{12}^q alone, instead one needs in addition an enhancement of Γ_{12}^s compared to its SM value by a factor of 2.8 up to 34. In the text it was discussed, however that Γ_{12}^s can not deviate more than about 30% from its SM value (neither due to new physics nor due to violations of quark hadron duality). On the other hand one should keep in mind that the central value measured by D0 is only about 2.5σ above the theoretical limit. To settle this problem clearly an independent measurement of semi leptonic CP asymmetries might shed light into the dark.

The success of the HQE for predicting inclusive processes in the b -system, raises of course again the question of the applicability of the HQE to the charm system. Which seems now at first sight to be more promising, because for D -mixing the energy release is not so different from $\Delta\Gamma_s$: $m_D - 2m_K \approx 0.9$ GeV and $m_D - 2m_\pi \approx 1.6$ GeV. Of course the situation is much more subtle because of the huge GIM cancellations in D -mixing, see e.g. ¹⁰³.



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